Joint conditionality in testing the beta-return relationship: Evidence based on the UK stock market

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Abstract

This paper examines the role of beta in explaining security returns in the UK stock market over the period of 1980–2006. The conditional relationship between beta and returns is examined. Conditional covariances and variances used to estimate beta are modeled as an ARCH/GARCH process, and once estimated, the beta-return relationship is tested conditionally upon the sign of the excess market return. The implication of the sign of the excess market return follows Pettengill et al. (1995). The main contribution of this paper is that, unlike other studies, a joint conditionality is applied to test the beta-return relationship. The results show the importance of recognizing the sign of the excess market return when testing the beta-return relationship, for only then is beta found to be a significant risk measurement. The premium payment found with respect to beta represents risk payments.

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1. Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) has been one of the most commonly adopted and tested models in the financial world. The CAPM claims that the expected return on any security can be explained solely by its level of systematic risk, as measured by beta. Beta is the only variable to explain the expected return on a security and the relationship that exists is a positive one. Early tests of the unconditional CAPM, were conducted by Lintner (1965), Black et al. (1972) and Fama and MacBeth (1973), all of which produced results in support of the central proposition of the CAPM, namely a positive risk-return relationship. Despite this early success for the

The CAPM is a model that is based upon expectation, however given the absence of expected data, the CAPM is tested using realized returns with the critical assumption applied being that realized returns accurately reflect, and thus can proxy for, expected returns. According to the CAPM there exists a positive relationship between beta and expected returns, this itself implies that the expected return on the market must always exceed the risk-free rate, i.e. the expected market risk premium is always positive. Given that the CAPM is tested using realized data, there is every possibility that at times the realized market risk premium may be negative. Pettengill et al. (1995), from here on referred to as Pettengill et al. (1995) recognizing the dilemma of using realized returns to test a model based on expectations, developed an alternative method for testing whether beta is a significant risk factor. All investors recognize that there is a nonzero probability that the return on the market will at times be less than the risk-free rate, if this was not the case no rational investor would ever invest in risk-free assets. Accepting this rationality, the question arises as to what should the risk–return relationship be when the excess market return is negative. Pettengill et al. (1995) infer that when the realized return on the market exceeds the risk-free rate (up markets) there exists a positive relationship between beta and returns, and when the realized market return is negative (down markets) the beta-return relationship should be negative. Pettengill et al. (1995) thus examined the role of beta conditional upon the sign of the realized market risk premium.

Pettengill et al. (1995) found, when applying their pioneering methodology, a significant conditional relationship between beta and returns in the US market over their entire sample period, subperiods, and also when the data was split according to the month of the year. A number of studies adopting a similar methodology followed. Studies on the UK market by Fletcher (1997), Hung et al. (2004) and Morelli (2007) found a significant conditional relationship between beta and returns. Isakov (1999) on examining the Swiss stock market, Faff (2001) on the Australian market, Ho et al. (2006) on the Hong Kong market, Elsas et al. (2003) on the German stock market, all found a significant conditional relationship between beta and returns.

This paper applies the Pettengill et al. (1995) methodology, in terms of testing the significance of beta conditional upon the sign of the realized market risk premium, though differs in terms of the method by which beta is estimated. The study by Pettengill et al. (1995), and subsequent studies adopting their methodology, estimated time varying betas by rolling regressions. The difference with respect to this paper is that time varying betas are estimated from changing conditional variances and covariances modeled using various versions of the autoregressive conditional heteroscedastic (ARCH) models. Variations in risk have tended to be modeled using Engle’s (1982) ARCH model and also Bollerslev’s (1986) GARCH model. Studies by Bollerslev et al. (1988), Bodurtha and Mark (1991), Ng (1991), and Hansson and Hordahl (1998) allowed time variations over covariances and variances in testing the conditional CAPM, adopting various versions of the family of ARCH models to model variances and covariances. Bollerslev et al. (1988) on examining the US market found conditional covariances varied over time and were shown to be significant determinants of time varying risk premium, thus supporting the conditional CAPM. The studies by Bodurtha and Mark (1991) and Ng (1991) both examining the US market, also produced evidence in support of the conditional CAPM, finding

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1 One must be clear that this is an inference made by Pettengill et al. (1995) for the CAPM is a model that attempts to explain the relationship between expected returns and beta and not realized returns, and does not imply what the relationship between beta and returns should be when the realized market risk premium is negative.

2 Increasing evidence suggests that expected return and risk vary over time, and thus the CAPM should incorporate such variations. This is the concept underlying the conditional CAPM. The development of a conditional CAPM is of great interest given that Hansen and Richards (1987) showed that in terms of testing the CAPM, tests incorporating changing conditional moments will tend to be more powerful. Hansen and Richards (1987) illustrated that failing to include conditional information in tests of the CAPM can lead to incorrect conclusions being made with respect to the conditional mean-variance efficiency of the market portfolio.
significant time variability in the conditional expected return and risk.\textsuperscript{3} Such findings supporting the conditional CAPM encouraged, over the years, further empirical studies testing conditional asset pricing models.\textsuperscript{4} In this paper beta is estimated as the ratio of the conditional covariance between the residuals from an autoregressive model for each security portfolio return and market portfolio return, and the conditional variance of the residuals from an autoregressive model for the market portfolio return. By modeling both components of a security’s beta, namely the covariance and variance, as an ARCH/GARCH process, allows conditional information to be incorporated into the model. Having estimated beta, its significance, in terms of being a priced risk factor, is tested conditionally upon the sign of the excess market return, in accordance with the Pettengill et al. (1995) methodology. This paper thus examines the significance of beta in explaining security returns conditional on both econometric information, where variances and covariances are modeled using the family of ARCH models, and also on the sign of the realized market risk premium. As far as I am aware this joint conditionality has not been conducted before and thus this paper provides a contribution to the existing literature.

The empirical results of this paper support the importance of accounting for the sign of the excess market return when examining the significance of beta in explaining security returns. Results show that when the sign of the excess market return is ignored beta is found to be an insignificant risk factor. During periods when the excess market return is positive (negative) a significant positive (negative) relationship is found between beta and returns. The results also show that this significant conditional beta-return relationship is not the result of any abnormal relationship in any particular month of the year. The relationship is shown to be consistent throughout the year, implying no evidence of seasonality in the risk–return relationship. The results are found to be robust across different time periods and also robust to portfolio reconfiguration.

The paper is structured as follows. Section 2 describes the conditional risk–return relationship. Sections 3 and 4 present the data and methodology. Section 5 discusses the empirical results. Section 6 examines the issue of robustness of the results, with Section 7 concluding the paper.

2. The conditional relationship between risk and return

The conditional version of the CAPM can be shown as follows:

$$E(r_{it}|\Phi_{t-1}) = \beta_i|\Phi_{t-1}(E(r_{Mt}|\Phi_{t-1}))$$  \hspace{1cm} (1)

where $r_{it}$ is the excess return for security $i$ and $r_{Mt}$ is the excess return on the market portfolio, $E(\Phi_{t-1})$ is the expectation conditional on the information set $\Phi$ available at time $t-1$, $\beta_i$ is the beta coefficient of security $i$, a measure of systematic risk given by the following expression:\textsuperscript{5}

$$\beta_i|\Phi_{t-1} = \frac{\text{Cov}(r_{it}, r_{Mt}|\Phi_{t-1})}{\text{var}(r_{Mt}|\Phi_{t-1})}$$ \hspace{1cm} (2)

Tests of the conditional relationship between beta and returns depend upon the information set $\Phi$ available. Different tests can be conducted dependent upon how the information set $\Phi$ is defined. Let $I$ represent a subset of information from the information set $\Phi$, and this subset of information represents econometric information.\textsuperscript{6} The conditional risk–return relationship tested in this paper can thus be shown as follows:

$$E(r_{it}|I_{t-1}) = \beta_i|I_{t-1}(E(r_{Mt}|I_{t-1}))$$ \hspace{1cm} (3)

\textsuperscript{3} The results produced by Ng (1991) were found to be sensitive to the criteria adopted to create portfolios, namely according to size and beta, finding that using beta portfolios produced results supporting the CAPM.


\textsuperscript{5} Eqs. (1) and (2) show that this conditional version of the CAPM allows the excess return of security $i$ to be time varying due to time variations in the conditional variance of the return on the market portfolio, the conditional covariance between the security’s return and the return on the market, and also the excess return on the market portfolio.

\textsuperscript{6} Given that $I$ is a subset of $\Phi$ it will not contain as much information as $\Phi$, though one can argue that if the risk–return relationship holds conditionally on information $I$ it will hold on information $\Phi$. 
Let us assume that the return on security $i$ and the market can be modeled as an autoregressive process as given by Eqs. (4) and (5).

$$
rit = a_0 + \sum_{j=1}^{g} a_j r_{it-j} + \varepsilon_{it} \\
$$ (4)

$$
M = a_0 + \sum_{j=1}^{g} a_j r_{Mt-j} + \varepsilon_{Mt} \\
$$ (5)

both Eqs. (4) and (5) can be decomposed into their expected and unexpected components as shown by Eqs. (6) and (7) respectively.

$$
rit = E(rit | It-1) + \varepsilon_{it} \\
$$ (6)

$$
M = E(r_{Mt} | It-1) + \varepsilon_{Mt} \\
$$ (7)

The disturbance terms $\varepsilon_{it}$ and $\varepsilon_{Mt}$ from Eqs. (6) and (7) can be decomposed as shown in Eqs. (8) and (9).

$$
\varepsilon_{it}, \varepsilon_{Mt} = E(\varepsilon_{it}, \varepsilon_{Mt} | It-1) + \eta_{it} \\
$$ (8)

$$
\varepsilon_{Mt}^2 = E(\varepsilon_{Mt}^2 | It-1) + \eta_{Mt} \\
$$ (9)

The expectation part of Eqs. (8) and (9) represents the conditional covariance between $rit$ and $M$ and the conditional variance of $M$ respectively, thus the risk measurement beta can be expressed as follows:

$$
\beta_{i|I_{t-1}} = \frac{E(\varepsilon_{it}, \varepsilon_{Mt} | It-1)}{E(\varepsilon_{Mt}^2 | It-1)} \\
$$ (10)

The risk–return relationship to be tested conditionally on information $I$ is given by Eq. (11) where the expected return on a security is dependent upon time varying risk, where the conditional information is incorporated by modeling the components of risk as ARCH/GARCH processes.

$$
E(rit | It-1) = \left( \frac{E(\varepsilon_{it}, \varepsilon_{Mt} | It-1)}{E(\varepsilon_{Mt}^2 | It-1)} \right) (E(r_{Mt} | It-1)) \\
$$ (11)

In order to proceed with the estimation of beta as shown by Eq. (10) it is necessary to specify parametric models for the expectations appearing in both the numerator and the denominator. Each of the expectations, $E(\varepsilon_{it}, \varepsilon_{Mt})$ and $E(\varepsilon_{Mt}^2)$, will be a function of the econometric information available at time $t – 1$. It is assumed that the sequence of squares and cross-products from the return forecast error as obtained from Eqs. (4) and (5) can be represented by a simple autoregressive process. For both components of conditional beta, $E(\varepsilon_{it}, \varepsilon_{Mt})$ and $E(\varepsilon_{Mt}^2)$, it is assumed that they could follow an ARCH, GARCH and GARCH-in-Mean process, a model whereby conditional variances and covariances are allowed to change over time. All the heteroscedastic models are adopted in the estimation process with the best fit model being the one selected.

Once beta is estimated from Eq. (10), the relationship between beta and returns, conditional on the econometric information $I$, can be tested from the following cross-sectional regression of security returns against corresponding betas.

$$
ri = \alpha_0 + \gamma_1 \beta_i + \xi_i \\
$$ (12)

where $\alpha_0$ should equal zero and $\gamma_1$ is the market risk premium, which should be positive and statistically significant thus implying that beta is a significant risk measure.

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7 Engle’s ARCH model was extended by Bollerslev (1986) to the GARCH (Generalised autoregressive conditional heteroscedasticity) model. GARCH and ARCH models are similar, the difference being that the conditional variance is also a function of past conditional variances. A further extension is the GARCH-M model (GARCH-in-Mean) as developed by Engle and Bollerslev (1986) where the conditional variance is simply incorporated into the mean equation.
In order to examine the relationship between beta and realized returns conditional also on the sign of the excess market return, testing is modified by including a dummy variable in the cross-sectional Eq. (12), thus allowing periods of positive and negative excess market returns to be separated out. This is in accordance with the methodology of Pettengill et al. (1995). This is shown as follows:

\[ r_i = a_0 + \delta \gamma_i^+ \beta_i + (1 - \delta) \gamma_i^- \beta_i + \xi_i \]  

(13)

where \( \delta = 1 \) if \( r_{Mt} > 0 \) and \( \delta = 0 \) if \( r_{Mt} < 0 \). The positive and negative symbol (\( ^+ \) and \( ^- \)) implying a positive and negative excess market return. By incorporating a dummy variable into the cross-section regression allows for the existence of a negative realized market risk premium. Again, one would expect \( a_0 \) to equal zero and \( \gamma_i^+ (\gamma_i^-) \) to be positive (negative) and statistically significant, implying the significance of beta as a risk measure.

Applying the methodology of Pettengill et al. (1995) allows additional conditional information, namely the sign of the excess market return, in addition to the econometric information discussed earlier, to be incorporated when examining the role of beta as a significant risk factor. It is important to recognize that any tests adopting the Pettengill et al. (1995) methodology cannot be viewed as testing the CAPM, but more of a test showing the significance of beta, given that the Pettengill et al. (1995) approach focuses on the relationship between beta and realized returns and not expected returns.

3. Data

The sample consists of 300 securities listed continuously on the UK stock exchange over the period 1st January 1980 to 31st December 2006. The securities selected represent a total of 18 different sectors and are thus a fair reflection of the different industry groups that constitutes the market. Data is obtained from the London Share Price Database (LSPD). All monthly security returns are adjusted for dividends and stock splits. The LSPD simply defines monthly returns as \( \ln(P_t + d_t) - \ln P_{t-1} \) where \( P_t \) represents the security price at the end of month \( t \) and \( d_t \) the declared dividend at the end of month \( t \). The three-month T-Bill rate is used to proxy for the risk-free rate which in turn is converted into its monthly equivalent so as to have a similar frequency with security returns. Market equity is also obtained for each security and is defined as the market share price multiplied by the number of issued shares. A value-weighted average of all 300 securities is used to proxy for the market portfolio.

4. Methodology

Tests of the CAPM have often been performed using portfolios as opposed to individual securities given that individual security betas are subject to greater measurement errors than portfolio betas. Securities are ranked in descending order on the basis of their mean return and sorted into 20 portfolios of 15 securities each. Table 1 shows summary statistics, namely mean, standard deviation, skewness and autocorrelations at various lags, for each of the 20 portfolios and the market portfolio. The skewness statistic is not large, though clearly shows that the returns are negatively skewed. Significant autocorrelations at differing lags (1, 2, 3, 6 and 12 lags) are detected for many of the portfolios, reducing in significance as the lag period increases.

Having formed the portfolios it is then necessary to model the time-series data for portfolio returns. Portfolio returns and market returns are modeled as an autoregressive process. Taking monthly returns for each of the 20 portfolios and also on the market portfolio over the total period January 1980 to

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8 The requirement in terms of the data used in this paper is that the companies selected must have been trading on January 1980 and still trading on December 2006. Such a selection excludes companies that have de-listed, have failed or have been taken over, and as a result of this there is a natural survival bias in the sample used. Any companies with missing observations were also excluded. The problem associated with thin trading is dealt with in part by the use of monthly returns as oppose to weekly or daily returns and also by excluding heavily illiquid securities. Given these restrictions, from a large data set of companies listed on the UK stock exchange, 306 companies did meet these requirements, of which 300 were used in this paper so as to create equal size portfolios.

9 This is important to point out so as to be clear in that the results obtained and conclusions drawn are not specific to any particular industry sector.
December 2006, an autoregressive model is fitted. With respect to the market portfolio, the square of the error term from the autoregressive model is then modeled using ARCH/GARCH models so as to estimate the conditional variance. The same is also applied to the cross-product from the error term fitted to the market portfolio and each individual portfolio of securities, thereby estimating the conditional covariances. The log-likelihood value and serial correlation statistic of residuals (Box-Ljung Q-statistic) is adopted to determine the best fit autoregressive model, and also the best fit ARCH/GARCH model, with the additional Lagrange multiplier test applied to test for the presence of conditional heteroscedasticity.10 Having estimated both the conditional covariances and the conditional variances, the conditional portfolio beta is estimated from Eq.(14).

\[
\beta_p|I_{t-1} = \frac{E(\epsilon_{pt}, \epsilon_{Mt}|I_{t-1})}{E(\epsilon_{Mt}^2|I_{t-1})}
\] (14)

As a consequence of the various lags in the ARCH/GARCH modeling some data is inevitably lost, and as a result the betas are estimated for all portfolios from July 1980 to December 2006 resulting in a total of 318 betas for each portfolio, one for each month.11

Having estimated beta, a cross-sectional regression, as shown in Eq.(15) is performed in order to determine whether beta is a priced risk variable.

\[
r_p = a_0 + \gamma_1 \beta_p + \xi_p
\] (15)

The pricing relationship is tested over the same time period over which beta was estimated, thus a total of 318 cross-sectional regressions are performed, one for each month in the testing period.

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10 The Lagrange multiplier test applied to test for the presence of conditional heteroscedasticity involves regressing the squared residuals from the best fit AR(q) model on q lagged values. The corresponding R² multiplied by the number of observations follows a χ² distribution.

11 With respect to the market portfolio the best fit model is found to be AR(3) with an additional autoregressive term at lag 6. Due to this lag the first 6 months of data are lost in the beta estimation stage, thus explaining why the betas are estimated from July 1980 onwards.

---

**Table 1**

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Mean</th>
<th>S.D.</th>
<th>Skewness</th>
<th>Autocorrelations at lags</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>0.0230</td>
<td>0.0382</td>
<td>−0.602</td>
<td>0.151</td>
</tr>
<tr>
<td>2</td>
<td>0.0221</td>
<td>0.0413</td>
<td>−0.584</td>
<td>0.129</td>
</tr>
<tr>
<td>3</td>
<td>0.0207</td>
<td>0.0361</td>
<td>−0.593</td>
<td>0.131</td>
</tr>
<tr>
<td>4</td>
<td>0.0191</td>
<td>0.0353</td>
<td>−0.431</td>
<td>0.116</td>
</tr>
<tr>
<td>5</td>
<td>0.0182</td>
<td>0.0396</td>
<td>−0.386</td>
<td>0.119</td>
</tr>
<tr>
<td>6</td>
<td>0.0173</td>
<td>0.0421</td>
<td>−0.410</td>
<td>0.123</td>
</tr>
<tr>
<td>7</td>
<td>0.0164</td>
<td>0.0371</td>
<td>−0.487</td>
<td>0.131</td>
</tr>
<tr>
<td>8</td>
<td>0.0159</td>
<td>0.0349</td>
<td>−0.597</td>
<td>0.152</td>
</tr>
<tr>
<td>9</td>
<td>0.0153</td>
<td>0.0375</td>
<td>−0.389</td>
<td>0.136</td>
</tr>
<tr>
<td>10</td>
<td>0.0147</td>
<td>0.0338</td>
<td>−0.352</td>
<td>0.121</td>
</tr>
<tr>
<td>11</td>
<td>0.0141</td>
<td>0.0428</td>
<td>−0.439</td>
<td>0.139</td>
</tr>
<tr>
<td>12</td>
<td>0.0135</td>
<td>0.0408</td>
<td>−0.604</td>
<td>0.167</td>
</tr>
<tr>
<td>13</td>
<td>0.0127</td>
<td>0.0364</td>
<td>−0.448</td>
<td>0.104</td>
</tr>
<tr>
<td>14</td>
<td>0.0121</td>
<td>0.0417</td>
<td>−0.498</td>
<td>0.147</td>
</tr>
<tr>
<td>15</td>
<td>0.0115</td>
<td>0.0341</td>
<td>−0.519</td>
<td>0.132</td>
</tr>
<tr>
<td>16</td>
<td>0.0096</td>
<td>0.0385</td>
<td>−0.486</td>
<td>0.128</td>
</tr>
<tr>
<td>17</td>
<td>0.0093</td>
<td>0.0336</td>
<td>−0.397</td>
<td>0.145</td>
</tr>
<tr>
<td>18</td>
<td>0.0089</td>
<td>0.0395</td>
<td>−0.601</td>
<td>0.109</td>
</tr>
<tr>
<td>19</td>
<td>0.0083</td>
<td>0.0316</td>
<td>−0.624</td>
<td>0.116</td>
</tr>
<tr>
<td>20</td>
<td>0.0076</td>
<td>0.0311</td>
<td>−0.412</td>
<td>0.124</td>
</tr>
<tr>
<td>Market</td>
<td>0.0145</td>
<td>0.0381</td>
<td>−0.421</td>
<td>0.129</td>
</tr>
</tbody>
</table>

Summary statistics provided for each of the 20 portfolios comprising of 15 securities each in addition to the market portfolio based on monthly returns over the total data period January 1980 to December 2006.
The monthly estimates $\gamma_1$ are averaged ($\bar{\gamma}_1$) from which the following hypothesis is tested: $H_0: \bar{\gamma}_1 = 0$ against the alternative $H_1: \bar{\gamma}_1 \neq 0$. The testing period runs from July 1980 to December 2006. To provide more robust tests, the testing period is also split into two equal subperiods running from July 1980 to September 1993, and October 1993 to December 2006.

To test the conditional relationship between beta and returns jointly conditional upon the sign of the realized excess market return, the cross-sectional regression shown by Eq. (16) is performed according to the methodology of Pettengill et al. (1995). The data is segmented between up and down markets in recognition of the ex-post negative excess market return.

$$rp = a_0 + \delta \gamma_1^* \beta_p + (1 - \delta) \gamma_1^- \beta_p + \xi_p$$

The risk premium, $\gamma_1$, is estimated during up and down markets. One would expect the risk premium to be positive during up markets ($\gamma^*$), and negative during down markets ($\gamma^-$). Monthly estimates $\gamma_1$ are averaged ($\bar{\gamma}_1$) from which the following hypotheses are tested: $H_0: \bar{\gamma}_1^* = 0$ against the alternative $H_1: \bar{\gamma}_1^* > 0$, and $H_0: \bar{\gamma}_1^- = 0$ against the alternative $H_1: \bar{\gamma}_1^- < 0$. Pettengill et al. (1995) point out that for a positive risk–return relationship to exist the excess market return should on average be positive and the risk premium in up and down markets should be symmetrical. Thus, the hypotheses tested are: $H_0: R_M - R_f = 0$ against the alternative $H_1: R_M - R_f \neq 0$, and $H_0: \bar{\gamma}_1^* - \bar{\gamma}_1^- = 0$ against the alternative $H_1: \bar{\gamma}_1^* - \bar{\gamma}_1^- \neq 0$. The symmetry hypothesis is tested by a two-population $t$-test, with the sign of the coefficients in down markets reversed and average monthly estimates recalculated.

5. Results

For each of the 20 portfolios the best fit autoregressive model, selected according to the log-likelihood value and Box-Ljung Q-statistic, is found to be either an AR(1), AR(2) or AR(3) process, with the market portfolio modeled by an AR(3) process with an additional autoregressive term at lag 6. Examination of the Box-Ljung Q-statistic at the sixteenth order for the best fit autoregressive model for all portfolios shows no real sign of any autocorrelation among the residuals. An autoregressive process is thus required to produce an uncorrelated sequence from the return series. If the residual series is strict white noise the squared residual should show no significant autocorrelation. The Box-Ljung Q-statistic for the squared residuals at the sixteenth order shows statistical significance in the squared residual series.\(^{12}\) The residuals may well be uncorrelated but they are not independent.\(^{13}\)

Beta estimation requires both the conditional variance and the covariance, both of which are modeled as an ARCH/GARCH process. With respect to the conditional variance of the market portfolio, Table 2 reports the best fit heteroscedastic model, namely GARCH (1,1).\(^{14}\) The best fit model is selected in accordance with the log-likelihood value. The Lagrange multiplier test for the presence of conditional heteroscedasticity is also reported (10.38, implying statistical significance at the 5% level) and shows that the null hypothesis of no ARCH errors is rejected. The GARCH (1,1) model is well specified, with the Box-Ljung Q-statistic at the sixteenth order showing no significant evidence of autocorrelation in the GARCH residuals.

With regards to the best fit ARCH/GARCH model for modeling the conditional covariance between each individual portfolio of securities and the market portfolio, the best fit model, based upon the log-likelihood value, is found to vary between ARCH (1) and ARCH (3) and GARCH (1,1). The Box-Ljung Q-statistic at the sixteenth order is found to show no sign of any significant autocorrelation in the ARCH residuals.\(^ {15}\)

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\(^{12}\) Box-Ljung Q-statistic for the residuals (residuals squared), for the 20 portfolios, ranges from 9.92 to 15.28 (27.39–41.18), and for the market portfolio 11.61 (32.41). The critical $\chi^2$ value at 16 degrees of freedom at 10% significance level is 23.54, thus the results show evidence of significant autocorrelation in the squared residuals. (Detailed results for all portfolios are available upon request).

\(^{13}\) If the null hypothesis that the squared residuals are uncorrelated is rejected, this would be equivalent to rejecting the hypothesis of no ARCH/GARCH errors.

\(^{14}\) The autoregressive process of the market portfolio is included in the full heteroscedastic process.

\(^{15}\) Box-Ljung Q-statistic for the residuals for the 20 portfolios ranges from 9.86 to 13.45. (Detailed results for all portfolios are available upon request).
Table 2
GARCH model estimates for return on the market portfolio.

<table>
<thead>
<tr>
<th></th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional mean</td>
<td>0.0125***(3.05)</td>
<td>0.1638**(2.72)</td>
<td>0.0419(0.81)</td>
<td>0.1467**(2.31)</td>
<td>0.1523*(1.91)</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\delta_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional variance</td>
<td>0.0011*(1.84)</td>
<td>0.3128***(2.903)</td>
<td>0.2073**(2.063)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>306.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM Test</td>
<td>10.38**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q (16)</td>
<td>15.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table reports the estimated coefficients with corresponding t-statistics shown in parentheses of an AR(3) with an additional autoregressive term at lag 6 and a GARCH (1,1) model for the market portfolio.

* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

Table 3
The relationship between beta and returns.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td>0.0062 (0.91)</td>
<td>0.0073 (1.04)</td>
<td>0.0052 (0.85)</td>
</tr>
</tbody>
</table>

The table reports the time-series average coefficients (risk premiums) over the testing periods obtained from monthly cross-section regressions (Eq.(15)) run on the 20 portfolios on the estimated betas, along with the t-statistics as shown in parentheses.

Having estimated both the conditional variance and the conditional covariance, beta is then estimated in accordance with Eq. (14) for each month, from which one can test whether beta is a priced risk variable. Table 3 reports the results from the cross-sectional regression as given by Eq. (15), showing the average risk premium over the total time period, \(\beta_1 = 0.0062\), and also for the two equal subperiods, \(\beta_1 = 0.0073\) and \(\beta_1 = 0.0052\). Over the total time period and also for both subperiods the results clearly show that although the risk premium is positive, implying at least an upward sloping risk–return relationship, it is not statistically significant, \(t = 0.91\) over the total time period, \(t = 1.04\) for the first subperiod, and \(t = 0.85\) for the second subperiod. The null hypothesis \(H_0: \beta_1 = 0\) cannot be rejected over all time periods tested. Beta plays no significant role in explaining security returns.16 Such findings are consistent with studies on the US markets by Davis (1994) and Fama and French (1992). Pettengill et al. (1995) argue that the finding of an insignificant beta can be explained by the aggregation of data during periods when the excess market return is both positive and negative. This is due to the fact that realized returns as opposed to expected returns are used to test the risk–return relationship, and given this, the beta-return relationship will be conditional on the sign of the excess market return.

Analysis of the market return data used in this paper finds that for 120 months (37% of the data) the excess market return is negative.17 Table 4 reports the results from testing the beta-return relationship conditional on the sign of the excess market return (Eq.(16)), reporting the average risk premium in both up and down markets. The results from the cross-sectional regression (Eq.(16)) clearly show a significant positive (negative) relationship between beta and returns during up (down) markets, consistent with the inference made by Pettengill et al. (1995). The null hypothesis of no beta-return relationship \(H_0: \beta_1^+ = 0\) and \(H_0: \beta_1^- = 0\) is clearly rejected. The mean value of the regression coefficients \(\beta_1^+ = 0.0189\) which is statistically significantly different from zero at the 1% level of significance \((t = 2.76)\). Such a finding implies that during up markets high beta portfolios exhibit higher returns than

16 The coefficient over the total time period \(\beta_1 = 0.0062\) equates to a monthly estimated market risk premium on beta of 0.62%. Although the risk premium is positive, implying a positive reward for exposure to market risk, this positive relationship is not statistically significant, thus implying that there does not exist in the UK stock market a significant risk premium on beta.
17 The study by Pettengill et al. (1995) reports a negative excess market return in 42% of their market data.
Table 4
The relationship between beta and returns during up and down markets.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Up markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_1^+$</td>
<td>0.0189 (2.76)**</td>
<td>0.0198 (3.58)**</td>
<td>0.0179 (2.21)**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Down markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_1^-$</td>
<td>-0.0197 (-3.11)**</td>
<td>-0.0206 (-3.34)**</td>
<td>-0.0188 (-2.93)**</td>
</tr>
</tbody>
</table>

The table reports the time-series average coefficients (risk premiums) in up and down markets over the testing periods obtained from monthly cross-section regressions (Eq. (16)) run on the 20 portfolios on the estimated betas, with $t$-statistics shown in parentheses. Significance tests with respect to beta are all based on a one-tailed test given that one assumes the sign of the coefficient.

* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

Table 5
Average monthly excess market return and test of symmetry hypothesis.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average excess market return</td>
<td>0.0081 (3.68)**</td>
<td>0.0073 (3.07)**</td>
<td>0.0089 (4.02)**</td>
</tr>
<tr>
<td>$\hat{\gamma}_1^+ = \hat{\gamma}_1^-$</td>
<td>0 (0.71)</td>
<td>0 (0.54)</td>
<td>0 (0.59)</td>
</tr>
</tbody>
</table>

The table reports the average excess market return and corresponding $t$-statistics shown in parentheses, testing whether on average it is found to be positive. The $t$-statistic from a two-population $t$-test is shown examining the symmetry hypothesis between the risk premium $\hat{\gamma}_1^+ - \hat{\gamma}_1^-$ in up and down markets.

* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

The mean value of the regression coefficients $\hat{\gamma}_1^-$ is $-0.0197$ which is also shown to be statistically significantly different from zero at the 1% level of significance ($t = -3.11$), supporting the claim that during down markets high beta portfolios earn a lower return than low beta portfolios. The findings suggest that beta risk is rewarded in up markets for losses incurred during down markets. For both subperiods the null hypothesis of no beta-return relationship during periods of both positive and negative excess market return is rejected. Thus the significant beta-return relationship holds across the total time period and also across both subperiods. Such findings are encouraging for beta as a risk measurement and supports the reasoning put forward by Pettengill et al. (1995) in explaining the insignificant relationship found between beta and returns, namely that it is due to the aggregation of positive and negative excess market returns.

As pointed out by Pettengill et al. (1995), a conditional beta-return relationship does not ensure a positive risk–return trade-off. Two conditions are required, namely a positive average excess market return and a consistent risk–return relationship during up and down markets. The results from testing both these requirements are shown in Table 5. The average monthly excess market return is found to be 0.81%, over the total time period, thereby rejecting the null hypothesis of zero excess return at the 1% level ($t = 3.68$). The results for both subperiods also reject the null hypothesis at the 1% level with an excess market return of 0.73% and 0.89% ($t = 3.07$ and $t = 4.02$ respectively). Such findings imply a significant positive reward for bearing market risk. With respect to the symmetry hypothesis, the risk premiums $\hat{\gamma}_1^+$ and $\hat{\gamma}_1^-$ from the cross-section Eq. (16) are examined. The results of a two-population $t$-test, as reported in Table 5 clearly show that over the total testing period, and both subperiods, one fails

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18 The coefficients during up and down markets, $\hat{\gamma}_1^+$ is 0.0189 and $\hat{\gamma}_1^-$ is $-0.0197$ respectively, represent the market risk premium. Both are found to be statistically significant in explaining average returns implying that, on recognition of the sign of the excess market return, beta produces a statistically and economically significant risk premium in the UK stock market. Such findings support the use of beta as a measure of market risk.
Table 6
The month by month relationship between beta and returns.

<table>
<thead>
<tr>
<th>Month</th>
<th>$\hat{\gamma}_1$ All markets</th>
<th>$\hat{\gamma}_1^{+}$ Up markets</th>
<th>$\hat{\gamma}_1^{-}$ Down markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>0.0048 (0.41)</td>
<td>0.0182 (1.90)**</td>
<td>−0.0201 (−2.58)*****</td>
</tr>
<tr>
<td>February</td>
<td>0.0057 (0.67)</td>
<td>0.0206 (3.07)*****</td>
<td>−0.0196 (−2.37)****</td>
</tr>
<tr>
<td>March</td>
<td>0.0081 (0.59)</td>
<td>0.0204 (2.82)*****</td>
<td>−0.0219 (−3.01)*****</td>
</tr>
<tr>
<td>April</td>
<td>0.0053 (0.54)</td>
<td>0.0196 (2.62)*****</td>
<td>−0.0231 (−3.18)*****</td>
</tr>
<tr>
<td>May</td>
<td>0.0036 (0.58)</td>
<td>0.0198 (2.78)*****</td>
<td>−0.0192 (−2.41)*****</td>
</tr>
<tr>
<td>June</td>
<td>0.0071 (0.63)</td>
<td>0.0167 (2.29)*****</td>
<td>−0.0183 (−2.19)****</td>
</tr>
<tr>
<td>July</td>
<td>0.0076 (0.66)</td>
<td>0.0208 (3.02)*****</td>
<td>−0.0156 (−1.82)**</td>
</tr>
<tr>
<td>August</td>
<td>0.0064 (0.74)</td>
<td>0.0199 (2.52)*****</td>
<td>−0.0222 (−2.67)**</td>
</tr>
<tr>
<td>September</td>
<td>0.0063 (0.32)</td>
<td>0.0162 (2.08)**</td>
<td>−0.0167 (−1.89)****</td>
</tr>
<tr>
<td>October</td>
<td>0.0069 (0.46)</td>
<td>0.0187 (2.45)*****</td>
<td>−0.0195 (−2.51)*****</td>
</tr>
<tr>
<td>November</td>
<td>0.0071 (0.92)</td>
<td>0.0161 (2.06)**</td>
<td>−0.0172 (−1.78)**</td>
</tr>
<tr>
<td>December</td>
<td>0.0057 (0.53)</td>
<td>0.0193 (2.57)*****</td>
<td>−0.0232 (−3.06)*****</td>
</tr>
</tbody>
</table>

The table reports the monthly time-series average coefficients (risk premiums) and corresponding $t$-statistics, shown in parentheses, over the total testing period estimated from the cross-section regression Eq. (15) (all markets) and 16 (up and down markets) so as to examine the impact of seasonality on the risk–return relationship.

* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

The issue of seasonality affecting the beta–return relationship has always been a key issue. Pettengill et al. (1995) found a significant unconditional beta–return relationship only in the months of January and February, with a number of months reporting a negative relationship, though not statistically significant. Strong relationships found in particular months can bias results when tests are conducted over all months. The significant positive unconditional beta–return relationship found by Pettengill et al. (1995) over their total testing period is clearly influenced by seasonality given that it is not found to be consistent throughout the year. Given the important influence seasonality can have, seasonality in the risk–return relationship is tested by dividing the data according to months of the year and re-estimating Eqs. (15) and (16). Table 6 shows the results from examining the monthly seasonality between beta and returns for cross-sectional Eqs. (15) and (16). With respect to the results from cross-sectional Eq. (15), the relationship found is consistent from month to month, in that for all months the relationship is found to be positive, however for all months the null hypothesis of no risk–return relationship cannot be rejected, implying that no significant relationship is found in any month between beta and returns from cross-sectional Eq. (15). This is inconsistent to the findings of Pettengill et al. (1995) based upon US securities, where evidence of strong seasonality is found in the months of January and February. When the data is separated according to the sign of the excess market return, a significant positive (negative) relationship is found during up (down) markets for all months. The null hypothesis of no risk–return relationship is rejected for all months. The size of the monthly mean risk premium during up markets ranges from 0.0208 in July to 0.0161 in November, and for down markets ranges from −0.0232 in December to −0.0156 in July. In order to determine whether the mean monthly risk premiums are statistically different from each other during both up and down markets, $t$-tests are performed on the monthly mean risk premiums across all months. It is found that for both up and down markets, the monthly mean risk premiums are not significantly different from each other at the 5% level of significance.\(^{19}\) Overall the relationship found between beta and returns during up and down markets is thus shown to be robust to dividing the data in accordance to the month of the year. This is an important finding, for a varying relationship during the year would imply a seasonal premium, instead the results clearly show a consistent beta–return relationship throughout the whole year. The premium payment found with respect to beta represents\(^{19}\) to reject the null hypothesis of symmetry. The risk–return relationship during up and down markets is consistent. This is supportive of Pettengill et al. (1995) who found a symmetrical relationship in US security returns. Thus the conditions required for a positive risk–return relationship are met.

\(^{19}\) Detailed results across all months are available upon request.
Table 7
The relationship between beta and returns for different size portfolios.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A: Results based on analysis of 15 portfolios of 20 securities each</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_1$</td>
<td>0.0079 (1.06)</td>
<td>0.0087 (1.21)</td>
<td>0.0073 (1.01)</td>
</tr>
<tr>
<td>Up markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}^+_1$</td>
<td>0.0217 (3.08)***</td>
<td>0.0229 (3.41)***</td>
<td>0.0205 (2.75)***</td>
</tr>
<tr>
<td>Down markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}^-_1$</td>
<td>-0.0231 (-2.87)***</td>
<td>-0.0249 (-3.59)***</td>
<td>-0.0214 (-2.65)***</td>
</tr>
<tr>
<td>B: Results based on analysis of 25 portfolios of 12 securities each</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_1$</td>
<td>0.0053 (0.64)</td>
<td>0.0064 (0.86)</td>
<td>0.0058 (0.71)</td>
</tr>
<tr>
<td>Up markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}^+_1$</td>
<td>0.0201 (2.62)***</td>
<td>0.0216 (2.91)***</td>
<td>0.0185 (2.52)***</td>
</tr>
<tr>
<td>Down markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}^-_1$</td>
<td>-0.0176 (-2.97)***</td>
<td>-0.0194 (-3.41)***</td>
<td>-0.0159 (-2.54)***</td>
</tr>
</tbody>
</table>

The table reports the time-series average coefficients (risk premiums) in all markets (cross-section regression Eq. (15)) and up and down markets (cross-section regression Eq. (16)) over the testing period, with $t$-statistics shown in parentheses. Significance tests with respect to beta are all based on a one-tailed test given that one assumes the sign of the coefficient.

* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

The finding of a consistent beta–return relationship throughout the year is consistent with the findings of Pettengill et al. (1995) during up and down markets.

6. Robustness test: portfolio reconfiguration

The empirical tests and the results obtained are based upon 20 portfolios each consisting of 15 securities. The results are shown to be robust in terms of time period, given the consistency across both subperiods examined and also monthly analysis. To add further robustness to the results and conclusions of this paper, reconfiguration of the portfolios is undertaken so as to determine the robustness to changes in the portfolio configuration, in terms of both a smaller number of portfolios with more securities, and a larger number with less securities.

The securities are divided into 15 portfolios consisting of 20 securities, and also 25 portfolios of 12 securities.20 For each portfolio, beta is estimated and empirical tests undertaken in accordance with the methodology discussed in Section 4. Table 7 reports the results.21 The results are consistent with those reported in Tables 3 and 4, in that once again a significant relationship between beta and returns is only found when recognition is given to the sign of the excess market return. The relationship, consistent with the inference made by Pettengill et al. (1995), is positive during up markets and negative during down markets. Across the total time period and also over both subperiods, the null hypothesis of no beta–return relationship is rejected at the 1% significance level. The empirical evidence clearly shows that irrespective of the size of the portfolios, there is strong support for a consistent significant relationship between beta and returns conditional on the sign of the excess market return. The results reported in Tables 3 and 4 are robust.

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20 Reducing the number of portfolios to less than 15 would question the reliability of the statistical tests.
21 Summary statistics for the 15 and 25 portfolios (portfolio mean, S.D., skewness, autocorrelations), and statistics for best fit AR and ARCH/GARCH model for each portfolio is available upon request.
7. Conclusion

This paper contributes to the existing literature regarding the role of beta in explaining security returns by applying a joint conditionality, by incorporating both various versions of Engle's (1982) ARCH models to estimate time varying betas and Pettengill et al. (1995) methodology, in examining the conditional relationship between beta and returns. A portfolio's beta is the ratio of the conditional covariance between the residuals from an autoregressive model for each portfolio return and the market portfolio return, and the conditional variance of the residuals from an autoregressive model for the market portfolio return. The conditional variance and covariance are modeled as an ARCH/GARCH process. Thus time varying betas are estimated from changing conditional variances and covariances. Pettengill et al. (1995) methodology allows the relationship between beta and returns to be tested upon the condition of the sign of the excess market return. Thus in this paper a joint conditionality is applied to test the beta-return relationship.

The results clearly show the importance of recognizing the sign of the excess market return when testing the relationship between beta and returns. Only when the relationship is tested with the additional condition based upon the sign of the excess market return, is beta found to be a significant risk factor, with a positive (negative) relationship with returns in up (down) markets. This is the case when the total data period is tested and also when divided into two equal subperiods implying a degree of robustness. The relationship is shown to be consistent throughout the year with no evidence found of monthly seasonality. Further robustness test provided further support for beta with findings showing that the beta-return relationship is robust to the reconfiguration of the portfolios. Clearly, the results and conclusions drawn from this paper apply specifically to the data and data period adopted in this study.

The results obtained from this paper based on UK securities are applicable to a degree to US studies. It is found that time varying variances and covariances can be modeled using ARCH/GARCH models, similar to US studies by Bollerslev et al. (1988), Bodurtha and Mark (1991) and Ng (1991), however unlike these studies which find evidence in support of a conditional CAPM, beta is found to be an insignificant risk measurement in the absence of recognition of the sign of the excess market return. However, the finding of a significant positive (negative) relationship between beta and returns during up (down) markets is consistent with Pettengill et al. (1995) study based on US securities, as to is the finding that the excess market return on average is positive and the risk premium in up and down markets is symmetrical. Pettengill et al. (1995) also report a consistent beta-return relationship throughout the year during both up and down markets, similar to the findings from this paper in which no evidence of seasonality is found.

It is found that 37% of the data exhibits monthly market returns less than the risk-free rate, and the results from this paper clearly show that it is due to the aggregation of positive and negative excess market returns that explains the failure to find any statistically significant relationship between beta risk and returns, for once these are accounted for beta is indeed found to be a significant measurement of risk.

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References


